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Singular behaviour of electrons and of composite fermions in a finite effective field

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Abstract. We calculate the self-energy $\Sigma_n(\epsilon)$ of fermions in Landau level n, in a finite field. Two cases are considered, in which fermions couple either to gauge fluctuations (as in the composite fermion gauge theory) or to phonons, as an example of a Fermi liquid. Perturbative calculations of the composite fermion spectrum show an unphysical suppression of the quasiparticle spectral weight at the composite fermion levels. We argue that this problem might be resolved by a non-perturbative calculation; alternatively, the system might be unstable.

The original Laughlin theory [1] of the fractional quantum Hall effect explains the ground state and low energy excitations for filling fractions $\nu = 1/(2k+1)$, with k a positive integer. Recent experiments [2] indicate a more general theory is needed to explain both the gapless state at $\nu = 1/2$, and the hierarchy of states for a general ν. A promising candidate involves new quasiparticles [3] called 'composite fermions' (CFs). CFs can be viewed as ordinary electrons, to each of which is attached an artificial flux tube, containing two flux quanta, oriented oppositely to the applied field $\boldsymbol{B}$. The net mean field $\boldsymbol{b}$ acting on the CFs becomes $b = B - \bar{B}_{1/2r}$, where $\bar{B}_{1/2r} = 4\pi r c \rho_e / e$ is the mean field from the artificial flux, and ρ_e is the mean electron density; we see that $b = 0$ when $\nu = 1/2r$. The set of FQHE states at $\nu = p/(1+2rp)$, with $p = 0, \pm 1, \pm 2$, etc, then arises because b is such that an integer number $|p|$ of CF Landau levels is filled; the 'principal hierarchy' has $r = 1$, and, as $|p| \to \infty$, so $\nu \to 1/2$, the 'bare' CF gap $\tilde{\omega}_c = eb/\tilde{m}$ goes to zero (here $\tilde{m}$ is the 'bare' CF mass, ignoring CF interactions).

A field theory of CFs [4] demonstrates that CF interactions resemble the gauge interactions in the gauge theory of high-T_c superconductors [5, 6], and this has led to several calculations of the renormalized properties of the $\nu = 1/2$ state [4] and the FQHE state with fractions near $1/2$; the FQHE calculations have looked at the renormalized gap [4, 7], the current response [8], and thermodynamic properties like the compressibility and density [7], in perturbative studies of the gauge interaction. However, these gauge interactions have severe infra-red divergences; they contain a term [4]

$$D_{11}(q,\omega) = \frac{q}{\chi q^s - \mathrm{i}\gamma\omega} \tag{1}$$

where $3 \geqslant s \geqslant 2$; if the Coulomb interactions between the CFs are screened by bringing a conducting plate up to the 2d semiconductor, then $s = 3$, whereas $s = 2$ for the completely unscreened case. If the CF self-energy is calculated perturbatively, to first order in $D_{11}(q,\omega)$, then one finds, at $\nu = 1/2$, that at $T = 0$

$$\Sigma(p,\epsilon) \sim \begin{cases} (\mathrm{i}\Omega_0/\epsilon)^{1/3}\epsilon & s = 3 \\ \epsilon \log(\epsilon) & s = 2 \end{cases} \tag{2}$$

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for p near the $\nu = 1/2$ Fermi surface. Non-perturbative calculations have also been done of $\Sigma(\epsilon)$ and of the response functions using eikonal expansions [9], $1/N$ expansions [10], and renormalization group analysis [11]. A crucial feature of these, heavily emphasized in [9, 10] and also in a recent paper of Stern and Halperin [12], is that Ward identities force the correlation functions to be much less singular than the self-energy. The self-energy is often considered to be unphysical since it is not gauge invariant (which causes the $\nu = 1/2$ self-energy to be *infinite* [6] at any finite T). However, one can also argue that its *pole structure* is gauge invariant and therefore physically meaningful. This is assumed by Stern and Halperin [12], in their analysis of the $s = 2$ case, and certainly one naively expects $\Sigma(\epsilon)$ to be well behaved away from $\nu = 1/2$, at least if $\bar{\epsilon} \ll \tilde{\omega}_c$, when the unrenormalized gap $\tilde{\omega}_c$ cuts off the IR divergences.

In this paper we take a closer look at this question, by calculating the self-energy $\Sigma_n(\epsilon)$ perturbatively, for a fermion in Landau level n. This calculation is applied to both CFs and also to an ordinary Fermi liquid in an applied field. We find that $\Sigma_n(\epsilon)$ has a rather peculiar singular structure. A less singular structure exists even for Fermi liquids.

We start from the usual lowest order perturbative expression for a 2d electronic self-energy:

$$\Sigma_n(\epsilon) = \int \frac{\mathrm{d}^2 q}{(2\pi)^2} \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \mathrm{Im} U(q,\omega) \sum_{m=-n}^{\infty} |\Lambda_n^m(q)|^2 \times \left(\frac{1 + n_B(\omega) - n_f(n+m)}{\epsilon - (n+m)\tilde{\omega}_c - \omega + i\delta} + \frac{n_B(\omega) + n_f(n+m)}{\epsilon - (n+m)\tilde{\omega}_c + \omega + \mathrm{i}\delta} \right) \tag{3}$$

where n, m are Landau level indices, $n_f(r) = n_f((r + 1/2)\tilde{\omega}_c)$ is the Fermi distribution for the rth Landau level, n_B is the Bose distribution and $U(\omega, q)$ describes the relevant interaction fluctuation. For gauge fluctuations

$$U(q,\omega) \sim -\left| \frac{\boldsymbol{k}_f \times \hat{q}}{m} \right|^2 D_{11}(q,\omega) \tag{4}$$

whereas for Fermi liquids $U(q,\omega) \sim f^2 \chi(q,\omega)$, where $\chi(q,\omega)$ is the relevant fluctuation propagator and f is the relevant Landau parameter [13]. As a concrete example we shall choose the coupled electron–phonon Fermi liquid, for which

$$U(q,\omega) \sim 2\bar{g}^2 \left(\frac{q v_f}{m} \right) \frac{q c_s}{\omega^2 - q^2 c_s^2} \tag{5}$$

where c_s is the sound velocity and $\bar{g}$ a dimensionless coupling. Finally, $\Lambda_n^m(q)$ is the overlap matrix element between plane wave and Landau level states, given by [14]

$$|\Lambda_n^m(q)|^2 = \left(\frac{q^2 l_0^2}{2} \right)^m \mathrm{e}^{-q^2 l_0^2/2} \frac{(n+m)!}{n!} \left| L_n^{n+m} \left(\frac{q^2 l_0^2}{2} \right) \right|^2 \tag{6}$$

where L_n^{n+m} is a Laguerre polynomial and $l_0 = (hc/eb)^{1/2}$ is the Landau length.

We calculate $\Sigma_n(\epsilon)$ in a quasiclassical approximation [7, 8], assuming $N \gg 1$ filled Landau levels (for CFs, $N = |p|$). The $T = 0$ calculations can be done analytically, and for $kT \ll \tilde{\omega}_c$, one can expand about the $T = 0$ answers [7]; notice the CF theory is only meaningful if $kT \ll \tilde{\omega}_c$.

Consider first the CF gauge theory results; writing $\Sigma = \Sigma' - \mathrm{i}\Sigma''$, we find

$$\Sigma_n'(\bar{\epsilon}) = \frac{-\mathrm{sgn}(\bar{\epsilon} - N)\Sigma_n''(\bar{\epsilon})}{\sqrt{3}} + \frac{2K_s}{\sqrt{3}} \left(\sum_{m=0}^{\min(\lfloor \bar{\epsilon}, \lfloor N)} (\bar{\epsilon} - m)^{-\alpha} - \sum_{m=\max(\lfloor \bar{\epsilon}+1, \lfloor N+1)}^{\infty} (m - \bar{\epsilon})^{-\alpha} \right) \tag{7}$$

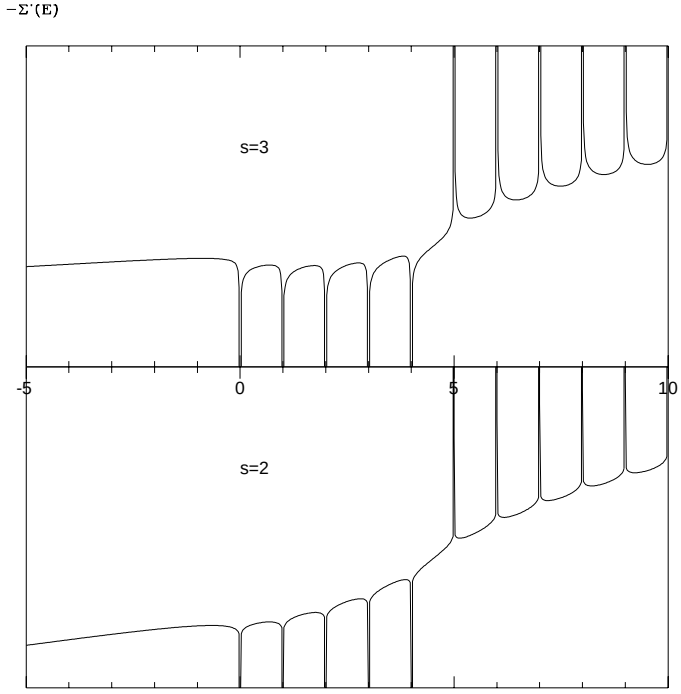

Figure 1. The real part $\Sigma_n(\epsilon)$ of the self energy for composite fermions shown for $s = 3$ and $s = 2$. We assume $p = 5$ (so that $\nu = 5/11$). The temperature is $kT = .03\tilde{\omega}_c$ and the chemical potential is pinned halfway between the fifth and sixth Landau levels.

$$\Sigma_n''(\bar{\epsilon}) = K_s\Bigg(\sum_{m=0}^{\lfloor\bar{\epsilon}}(n_B(\bar{\epsilon}-m)+n_f(N-m))(\bar{\epsilon}-m)^{-\alpha} + \sum_{\max(\lfloor\bar{\epsilon}+1,0)}^{\infty}(n_B(m-\bar{\epsilon})+n_f(m-N))(m-\bar{\epsilon})^{-\alpha}\Bigg) \tag{8}$$

for $s > 2$ and

$$\Sigma_n'(\bar{\epsilon}) = \frac{K_2}{\pi}\log|m-\bar{\epsilon}|\Bigg(-\sum_{m=0}^{\lfloor N}n_f(m-N)+\sum_{m=\lfloor N+1}^{\infty}n_f(N-m)\Bigg) \tag{9}$$

$$\Sigma_n''(\bar{\epsilon}) = K_2\Bigg(\sum_{m=0}^{\lfloor\bar{\epsilon}}(n_B(\bar{\epsilon}-m)+n_f(N-m))+\sum_{m=\max(0,\lfloor\bar{\epsilon}+1)}^{\infty}(n_B(m-\bar{\epsilon})+n_f(m-N))\Bigg) \tag{10}$$

for $s = 2$. In these equations K_s is a constant, $\bar{\epsilon} = \epsilon/\tilde{\omega}_c$, $\lfloor r$ is the greatest integer less than r, and the exponent $\alpha = (s-2)/s$ is positive. We show $\Sigma_n'(\epsilon)$ in figure 1 for $s = 2, 3$. When $s = 3$ there are inverse cube root divergences $|\bar{\epsilon}-r|^{-1/3}$ in $\Sigma'(\bar{\epsilon})$ as one approaches each Landau level, from either side. When $s = 2$ these become logarithmic divergences. However $\Sigma_n''(\epsilon)$ only shows divergences, for $s > 2$, when $\epsilon \to r\omega_c$ from *above*; it is finite if $\bar{\epsilon} = r + 0^-$. As $\tilde{\omega}_c \to 0$ the strength of these divergences vanishes and we are left with the smooth curves of equation (2) above.

An apparent pathology in these results is seen by calculating $z_n^{-1}(\epsilon) = 1 - \partial\Sigma_n'/\partial\epsilon$; one

then sees that as ϵ approaches $r\tilde{\omega}_c$, we get the divergence $z_n^{-1}(\bar{\epsilon}) \sim \pm|\bar{\epsilon} - r|^{-(1+\alpha)}$, i.e., an inverse power divergence. That the wave-function renormalization z_n^{-1} should diverge to $+\infty$ might have been expected; but it also shows an unnerving divergence to $-\infty$ near each Landau level.

Notice that a divergence in $|z_n^{-1}|$ is not unique to these singular interactions, only the sign. An equivalent calculation for the electron–phonon problem gives

$$\Sigma_n'(\bar{\epsilon}) = \frac{K_\phi}{\pi}\left(\sum_{m=\lfloor N+1}^{\infty} (m - \bar{\epsilon}) \log\left|\frac{\omega_D + m - \bar{\epsilon}}{m - \bar{\epsilon}}\right| + \sum_{m=0}^{\lfloor N} (\bar{\epsilon} - m) \log\left|\frac{m - \bar{\epsilon}}{m - \bar{\epsilon} - \omega_D}\right| \right) \tag{11}$$

at $T = 0$, with a trivial generalization to finite T. Here K_ϕ is a constant and ω_D is a Debye cut-off (i.e., $\omega_D = \theta_D/\tilde{\omega}_c$). In figure 2 we plot $\partial\Sigma'/\partial\epsilon$, derived from (11); the divergences now have the form $z_n^{-1}(\epsilon) \sim -\log|\bar{\epsilon} - r|$, and we still get regions with very large positive $z_n^{-1}(\epsilon)$. This result clearly has nothing to do with any lack of gauge invariance of $\Sigma_n(\epsilon)$.

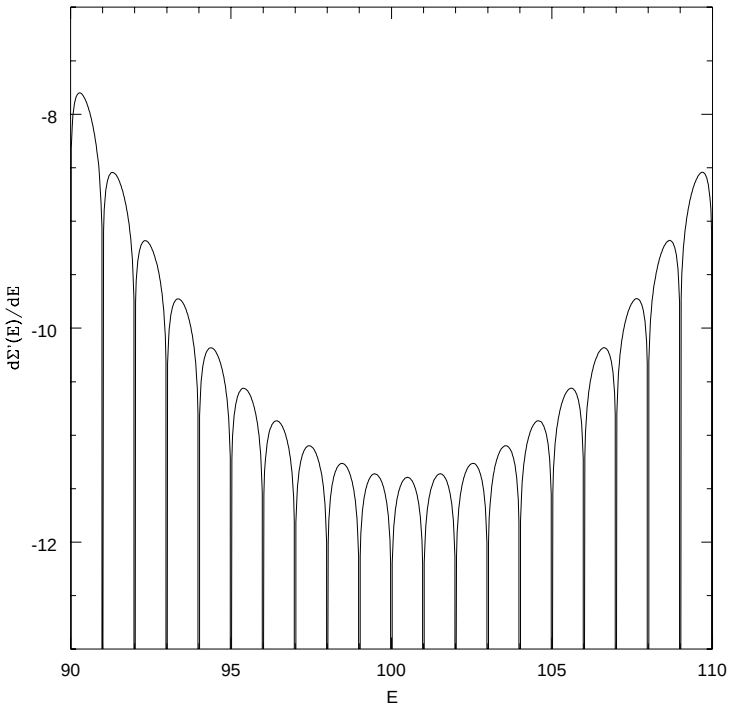

Figure 2. The derivative $\partial\Sigma'(\epsilon)/\partial\epsilon$ for a Fermi liquid (here an electron–phonon system) with 100 filled Landau levels, calculated at $T = 0$.

The mathematical origin of these results is as follows. First, we assumed a quasi-particle form for the internal fermion line in $\Sigma_n(\epsilon)$; in fact we assumed a form, for a system of unit area, given by

$$l_0^2 G_n(\epsilon) = \frac{1}{\epsilon - n\tilde{\omega}_c - \Sigma_n(\epsilon)} \sim \frac{z}{\epsilon - \tilde{n}\omega_c} \tag{12}$$

with z a constant renormalization factor. (Note that $G_n(\epsilon) \sim l_0^2$, in (12), because each Landau level has degeneracy l_0^{-2}.) Second, our vertex Λ_n^m has no structure arising from the interactions, since we are working in lowest-order perturbation theory. This explains

the divergence in $|z^{-1}|$, for both theories it comes from the massive degeneracy l_0^{-2} in each Landau level. From this point of view the positive divergence of z^{-1} in the electron–phonon case is no different in principle from what occurs for an Einstein phonon spectrum $\omega_q = \delta(\omega - \Omega_0)$.

On the other hand the novelty of the gauge theory result is that $\Sigma_n'(\epsilon)$ shows a positive divergence (for $\epsilon > n$) on both sides of the Landau level, thereby causing both positive and negative divergences in z^{-1}. Mathematically this arises because $\Sigma_n''(\epsilon)$ is essentially composed of a set of asymmetric peaks around $\bar{\epsilon} = r$, of the form $\sim (\bar{\epsilon} - r)^{-\alpha}\theta(\bar{\epsilon} - r)$, which have long tails for $(\bar{\epsilon} - r) \gg 1$. The Hilbert transform of such a function does *not* change sign as $\bar{\epsilon}$ crosses r, unless $\alpha > 1/2$; for the CF gauge theory $0 \leqslant \alpha < 1/3$. Thus the sign change in $z_n(\epsilon)$, each time ϵ crosses a Landau level energy $r\omega_c$, is caused by these long tails.

The consequences for the physical pole structure are interesting. The fermion spectral function $A_n(\epsilon)$, defined as usual in terms of $G_n(\epsilon)$ by

$$G_n(\epsilon) = \int_{-\infty}^{\infty} \mathrm{d}x \frac{A_n(x)}{\epsilon - x + \mathrm{i}\delta_x} \tag{13}$$

(where $\delta_x = \delta \mathrm{sgn} x$ and $\delta = 0^+$) goes to zero as $|\epsilon - r\omega_c| \to 0$; e.g., for $s > 2$ ($\alpha > 0$), one has

$$A_n(\epsilon) \to \left\{ \begin{array}{ll} \frac{6}{5\sqrt{3}}(\bar{\epsilon} - r)^{\alpha} & (\bar{\epsilon} > r) \\ c_r(r - \bar{\epsilon})^{2\alpha} & (\bar{\epsilon} < r) \end{array} \right\} \qquad \text{as } |\bar{\epsilon} - r| \to 0 \tag{14}$$

where c_r is a constant. This structure is both unphysical and in obvious contradiction with the form (12), assumed for the internal fermion lines—it makes no sense for the spectral weight to vanish on the Landau levels. One may also define a renormalized quasiparticle spectrum E_n, given by

$$E_n = n\tilde{\omega}_c - \Sigma_n'(E_n). \tag{15}$$

This shows a rather complicated structure when one is well away from the Fermi surface (i.e., $|n - p| \ll 1$), but for $|E_n| < \tilde{\omega}_c$ one finds a unique pole, which can be used to define a quasiparticle gap Δ which agrees with that found previously [7, 12]. However, the unphysical nature of $A_n(\epsilon)$ in (14), and its inconsistency with (12), brings these results into question.

One's first reaction to this, in the context of singular interactions, is that the theory is intrinsically IR divergent anyway, and that the way to cure divergences is to do a self-consistent calculation (just as in the zero-field case). There have been numerous investigations of vertex corrections in the case of zero applied field (no Landau levels), for both the gauge theories [7, 9–12] and the electron–phonon problem [15]. However, as far as we know, no equivalent investigations have been done for the finite-field problem, and in fact it is not completely obvious to us how the finite-field consistency problem can be resolved. On physical grounds we expect that some remnant of the sharp Landau level structure should survive interactions, i.e., that

$$l_0^2 G_n(\epsilon) \sim \frac{z_n}{\epsilon - E_n} + \text{incoherent} \tag{16}$$

near the renormalized energies E_n (otherwise we would not get a fractional quantum Hall effect). However, as we have just seen the large degeneracy at $\epsilon = E_n$ will tend to cause divergences in z_n^{-1}. The way to eliminate these is clearly through divergent vertex corrections to the 3-point vertex Λ, to cancel those in z_n^{-1}. An attempt to devise a self-consistent scheme of this kind is complicated by the IR divergences that already exist in the

theory as $\tilde{\omega}_c \to 0$, and it appears as though a non-perturbative formulation of the problem is necessary.

Another possibility is that there really is some kind of instability in the theory. In experiments one might expect this instability to be eliminated by impurity scattering, which leads to 'Dingle broadening' of the Landau levels, thereby destroying the sharp Landau level structure which leads to the divergence in $|z^{-1}(\epsilon)|$. In most experiments the Dingle temperature, T_D, which parametrizes this broadening, is greater than 100 mK, and the ratio $2\pi kT_D/\tilde{\omega}_c$ is rarely less than 0.1. Thus the very narrow divergent behaviour will be very difficult to see. However, we should note that we do not yet have a theory which combines the effects of gauge interactions and impurity scattering on the CFs, apart from perturbative results [4]. Although the experiments [2] are presently being analysed in terms of conventional dHvA or SdH expressions involving T_D (or a relaxation time), it is likely that a more realistic theory would involve at least an energy-dependent relaxation time, depending very rapidly on ϵ in the vicinity of the Landau levels.

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